IMPORTANT DERIVATIONS

(3 marks derivations)

1. Derive the mirror formula for concave mirror.

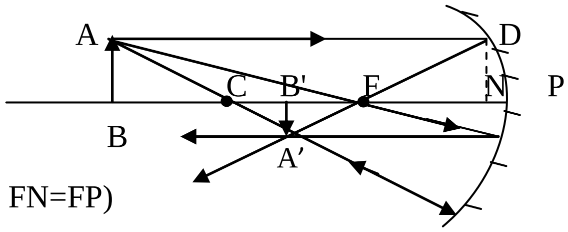
Let, P = pole and F=principal focus. Object AB is place beyond C and the real image formed by it is A'B' between F and C.

From similar triangles A'B'C and ABC

$$\frac{AB}{A'B'} = \frac{BC}{B'C} \quad .... \ (1)$$

From similar triangles A'B'F and DNF

$$\frac{DN}{A'B'} = \frac{FN}{B'F} \quad \text{or} \quad \frac{AB}{A'B'} = \frac{FN}{B'F} \quad ..... \ (2)$$



From (1) and (2)

$$\frac{BC}{B'C} = \frac{FN}{B'F} = = \frac{FP}{B'F}$$

Or, $\frac{PB-PC}{PC-PB'} = \frac{FP}{PB'-PF}$ (∵ the aperture is small, FN=FP)

Using sign convention, the above equation can be written as

$$\frac{-u-(2f)}{-2f-(-v)} = \frac{-f}{-v-(-f)}$$

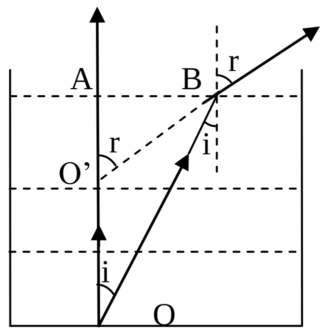
Or, $2f^2 - vf = -uf + uv + 2f^2 - 2f\,v$

Or, $f\,v + u\,f = u\,v$

Dividing both sides by uvf, we get

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$ which is the required formula.

2. Show that refractive index = real depth/apparent depth.



'O' is the object lying at the bottom of a tank containing water. O' is its virtual image.

From Snell's law, ${}^{w}n_a = \frac{\sin i}{\sin r} = \frac{AB/BO}{AB/BO'} = \frac{BO'}{BO}$

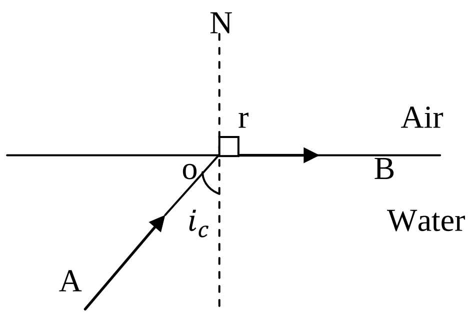
Since the aperture of the eye is very small, the two rays from A and B will enter the eye, only if B lies very close to A. Then, BO'≃AO' and BO≃AO.

∴ ${}^{a}n_{w} = 1 / {}^{w}n_a = \frac{AO}{AO'} = \frac{\text{Real depth}}{\text{Apparent depth}}$

3. Derive the relation between critical angle and refractive index of the medium.

When a ray of light passes through denser (water to a rarer (air) medium, then, according to Snell's law,

$${}^{w}n_{a} = \frac{\sin i}{\sin r}$$



when, $i = i_e$, $r = 90^o$

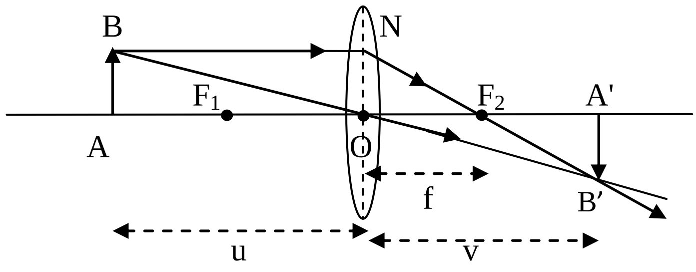
∴ ${}^{w}n_{a} = \frac{\sin i}{\sin r} = \frac{\sin i_c}{\sin 90°} = \sin i_c$

Or, $\boxed{{}^{a}\mathbf{n}_{w} = \frac{1}{\sin i_c}}$

** In general, $\mathbf{n} = \frac{1}{\sin i_c}$

4. Obtain lens formula for a thin convex lens when the image is real.

Let, O = optic centre, F_1 and F_2 be the principal focii. Object AB is place beyond F_2 and the real image formed by it is A'B' beyond F_2.



From similar triangles A'B' F_2 and ONF$_2$

$$\frac{A'B'}{ON} = \frac{F_2A'}{OF_2} \quad \text{or} \quad \frac{A'B'}{AB} = \frac{F_2A'}{OF_2} \qquad \ldots.\ (1)$$

From similar triangles A'B'O and ABO

$$\frac{A'B'}{AB} = \frac{OA'}{OA} \qquad \ldots..\ (2)$$

From (1) and (2)

$$\frac{F_2A'}{OF_2} = \frac{OA'}{OA}$$

Or,

$$\frac{OA' - OF_2}{OF_2} = \frac{OA'}{OA}$$

Using sign convention, the above equation can be written as

$$\frac{v-f}{f} = \frac{v}{-u}$$

On cross-multiplying, we get, v f = - u v + u f

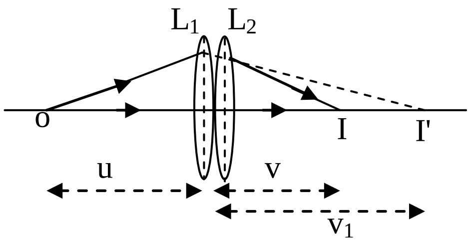
Dividing both sides by uvf, we get, $\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$ which is the required formula.

5. Two thin convex lenses of focal lengths f_1 and f_2 are kept in contact with each other coaxially. Deduce an expression for the effective focal length of the combination.

Let a point object 'O' is placed on the common principal axis. f_1 and f_2 be the focal lengths of lenses L_1 and L_2 .

In the absence of L_2, the image formed by L_1 will be at I_1.

So, $\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u}$ … (1)



In the presence of second lens L_2, I_1 will be the virtual object and the final image will be formed at I.

So, $\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v_1}$ … (2)

Adding equations (1) and (2), we get

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u} \qquad \ldots\ (3)$$

Now, if the lens combination is replaced by a single lens of focal length ' f ' , then

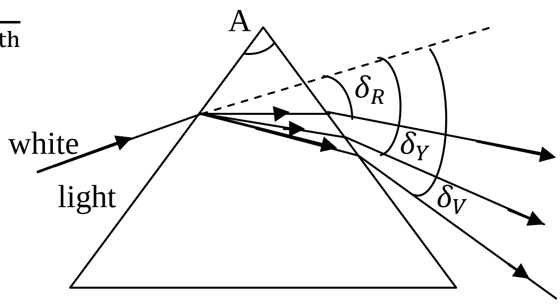
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \qquad \ldots\ (4)$$

From equations (3) and (4), $\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$ This is the required relation.

6. Derive an expression for dispersive power of a prism.

Dispersive power, **w** $= \frac{\text{angular dispersion}}{\text{deviation for the mean wavelength}}$

$$= \frac{\delta_V - \delta_R}{\delta_Y}$$



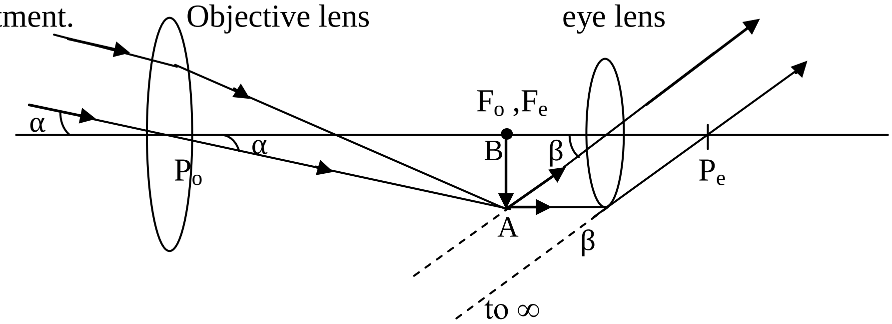
Substituting the values of δ_V & δ_R , we get,

$$\mathbf{w} = \frac{\{(n_V-1)A\} - \{(n_R-1)A\}}{(n_y-1)A} = \frac{n_V - n_R}{n_Y - 1}$$

7. What is meant by ‘normal adjustment’ in case of an astronomical telescope? With the help of a neat and labeled ray diagram, obtain an expression for the magnifying power of the telescope in normal adjustment.

Definition: - When the final image is formed by the telescope at infinity, then the telescope is said to be in normal adjustment.

Ray diagram:



Magnifying power = $\frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}$ which is the required relation.

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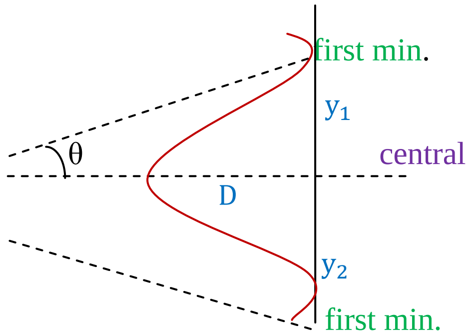
8. In the experiment on diffraction due to a single slit, show that the angular width of the central maximum is twice that of the first order secondary maximum.

The general maximum has between first minima on either side of the central maxima. We know that, If ‘d’ is the width of slit, for first minima,

$$d\sin\theta = \lambda \Rightarrow d\,\theta = \lambda \quad (\because \text{ for small angles, } \sin\theta \simeq \theta)$$

$$\tan\theta = \frac{y_1}{D}$$

$$\Rightarrow \quad \theta = \frac{y_1}{D}$$



max.

$$\Rightarrow \quad \frac{\lambda}{d} D = y_1 = y_2$$

Hence, whole width on secondary maxima on one side is $\frac{\lambda D}{d}$.

The angular width of the central maxima = $\frac{2\,D\,\lambda}{d}$

So, angular width of the central maxima is twice that of the first order secondary maximum.

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9. An unpolarised light is incident on the boundary between two transparent media. State the condition when the reflected wave is totally plane polarized. Find out the expression for the angle of incidence in this case.

Condition: The reflected ray is totally plane polarized, when reflected and refracted rays are perpendicular to each other. ∠BOC = 90°

If i_p is angle of incidence, r’ is angle of reflection and r is angle of refraction, then according to law of reflection

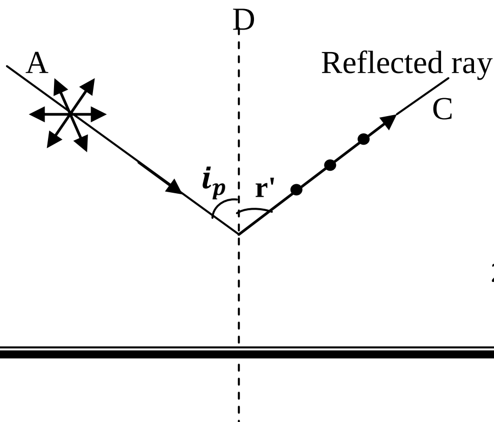
$$i_p = r'$$

and from fig. $r' + 90° + r = 180°$

$$\Rightarrow \quad i_p + r = 90° \quad \ldots \ (1)$$

$$\Rightarrow \quad r = (90° - i_p) \ \ldots. \ (2)$$

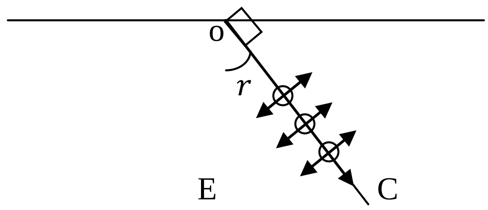
(incident ray)



From Snell's law, refractive index of second medium relative to first medium (air) say,

$$n = \frac{\sin i_p}{\sin r} = \frac{\sin i_p}{\sin(90° - i_p)} = \frac{\sin i_p}{\cos i_p} = tan i_p$$

∴ Angle of incidence, $\boldsymbol{i_p = tan^{-1}(n)}$



Refracted ray

10. Using Huygen's wave theory, verify the first law of reflection.

Let, XY be a reflecting surface. At t = 0, the wavefront touches the surface at 'A'. After time't', the point B of wavefront reaches the point C of the surface.

According to Huygen's principle, each point of wavefront acts as a secondary waves. The secondary wavelets from point A begins to spread in all directions and traverse distance AD (= v t) in time't'. In the same time't', the point B of wavefront, after travelling a distance BC, reaches point C. As the incident wavefront AB advances, the secondary wavelets starting from points between A and C, one after the other and will touch CD simultaneously. CD represents reflected wavefront.

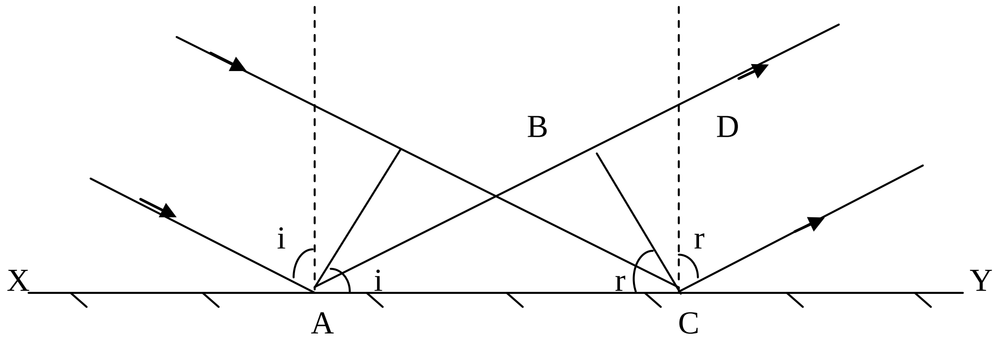
Now, in right-angled triangles ABC and ADC, ∠ABC = ∠ADC (both are = 90º)

Side BC = AD (both are = v t) and side AC is common.

So, both triangles are congruent.

∴ ∠BAC = ∠ACD

i.e., incident wavefront AB and reflected wavefront CD make equal triangles with the reflecting surface. Hence, angle of incidence = angle of reflection (which is the first law of reflection).



11. Using Huygen's wave theory, verify the Snell's law.

Suppose a plane wavefront AB in first medium is incident obliquely on the boundary surface XY and its end A touches the surface at A at time, t = 0 while the other end B reaches the surface at point B' after time-interval 't'. Clearly BC = v_1t. As the wavefront AB advances, it strikes the points between A and C of boundary surface.

According to Huygen's principle, secondary wavelets originate from these points, which travel with speed v_1 in the first medium and speed v_2 in the second medium.

Secondary wavelet starts from A, which traverses a distance AA' (= $v_2 t$) in second medium in time't'. In the same time-interval 't', the point of wavefront traverses a distance BC (= $v_1 t$) in first medium and reaches C, from where the secondary wavelet now starts. Clearly, BC = $v_1 t$

And AD = $v_2 t$.

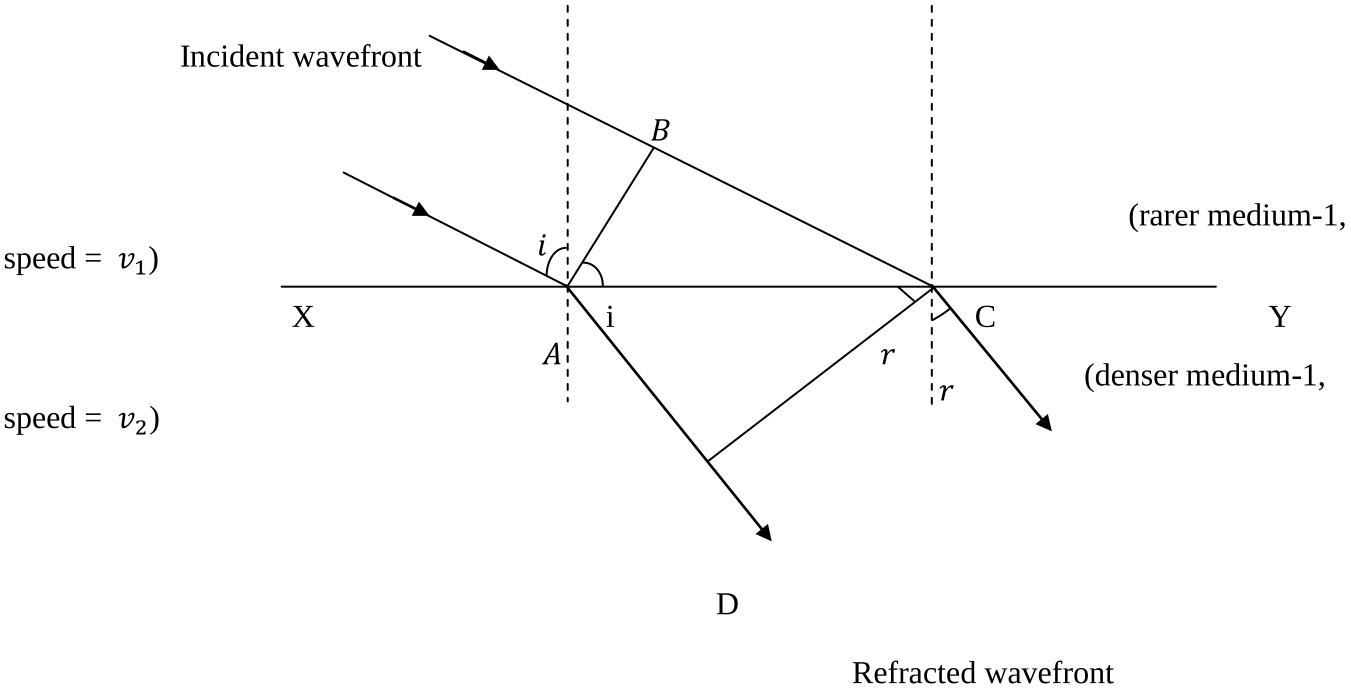
As the incident wavefront AB advances, the secondary wavelets start from points between A and C, one after the other and will touch CD simultaneously. According to Huygen's principle, CD is the new position of wavefront AB in the second medium. Hence, CD will be the refracted wavefront. Let the incident and refracted wavefronts make angles 'i' and 'r' respectively with refracting surface.

In right-angled triangle ACB, $\angle ABC = 90^o$ and $\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}$ …. (i)

Similarly, in right-angled triangle ADC, $\angle ADC = 90^o$ and $\sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}$ …. (ii)

Dividing equation (i) by (ii), we get, $\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = constant$ (refractive index of the medium).

This is the Snell's law.



(5 marks derivations)

12. Derive an expression for the refractive index of the material of prism.

Diagram shows section ABC of a prism taken by a vertical plane perpendicular to the edge. BC is base of the prism and AB & AC are its two refracting surfaces. PQ is incident ray, QR is refracted ray and RS is emergent ray.

In quadrilateral AQN_2R, $\angle AQN_2 + \angle ARN_2 = 180^o$ …. (1)

$\angle A + \angle QN_2R = 180^o$

In $\Delta\, QRN_2$, $\angle r_1 + \angle r_2 + \angle QN_2R = 180^o$ …. (2)

From equations (1) and (2), $\angle A = \angle r_1 + \angle r_2$ ….. (3)

In $\Delta\, XQR$, $\angle XQR = \angle i - \angle r_1$ & $\angle XRQ = \angle e - \angle r_2$

Since exterior $\angle TXR$ = interior $\angle XQR$ + interior $\angle XRQ$

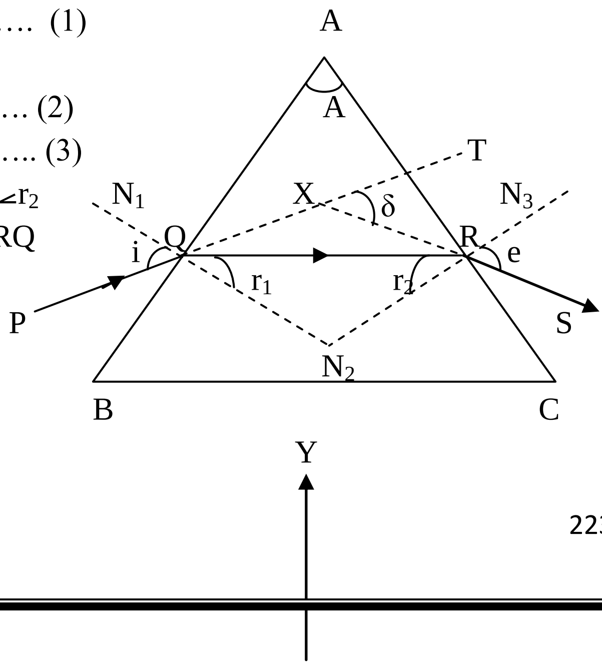
$\therefore \angle \delta = (\angle i - \angle r_1) + (\angle e - \angle r_2)$

$= (\angle i + \angle e) - \angle A$

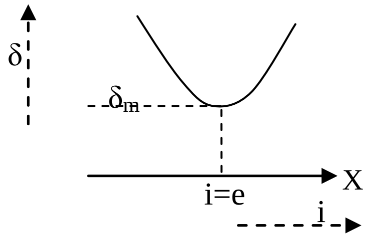
Or, $\angle A + \angle \delta = \angle i + \angle e$ …. (4)

A graph between $\angle i$ and $\angle\delta$ shows that,

$\angle\delta$ is more when $\angle i$ is either small or large.



$\angle\delta$ is minimum for some intermediate value of $\angle i$.
From graph, when $\angle\delta = \angle\delta_m$, then $\angle i = \angle e$ & $\angle r_1 = \angle r_2$
Now, from equations (3) and (4), we get,
$\angle A = 2r \Rightarrow r = \frac{\angle A}{2}$ & $\angle A + \angle\delta_m = \angle i + \angle i \Rightarrow \angle i = \frac{\angle A + \delta m}{2}$
From Snell's law, $n = \frac{\sin\left(\frac{\angle A+\delta m}{2}\right)}{\sin\left(\frac{\angle A}{2}\right)}$ This is the required expression.

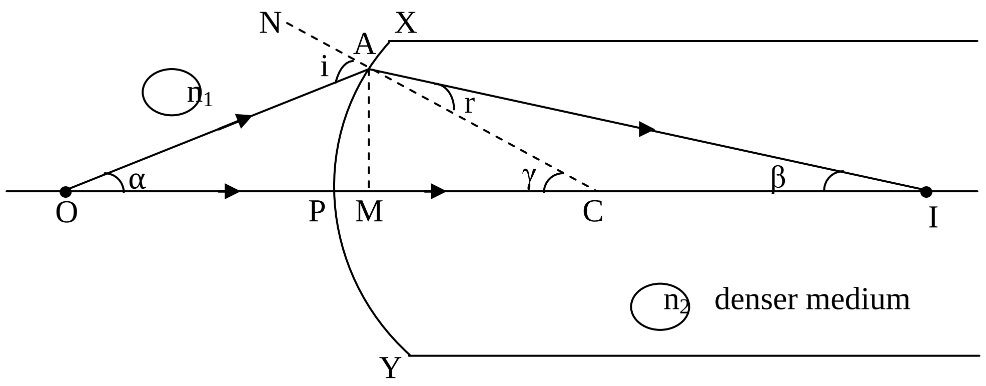


13. With the help of a ray diagram, show the formation of image of a point object by refraction of light at a convex spherical (convex) surface separating two media of refractive indices n_1 and n_2 ($n_2 > n_1$) respectively. Using this diagram, derive the relation $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$. Also write the sign conventions used and assumptions.

Sign Convention used:-
a) All the distances are measured from the pole.
b) The distances measured in the direction of incident light are taken as positive.
c) The distances measured in the direction opposite to the direction of light are taken as negative.

Assumptions:-
a) The aperture of the spherical refracting surface is small.
b) The object is a point object and lies on the principal axis.
c) The incident ray, the refracted ray and the normal to the spherical surface make small angles with the principal axis.

Let XPY = convex spherical refracting surface
O = point object in rarer medium
I = real image in denser medium



From ray diagram, from ΔAOC, $i = \alpha + \gamma$
From ΔAIC, $\gamma = r + \beta \Rightarrow r = \gamma - \beta$
According to Snell's law, $\frac{\sin i}{\sin r} = \frac{n_2}{n_1} \Rightarrow n_1 \sin i = n_2 \sin r$
Since the angles are small, $\therefore$ $n_1 i = n_2 r$ or, $n_1 \tan i = n_2 \tan r$
Substituting for i & r, in the above equation, we get

$$n_1 \tan(\alpha + \gamma) = n_2 \tan(\gamma - \beta)$$

Or, $n_1 \{\tan\alpha + \tan\gamma\} = n_2 \{\tan\gamma + \tan\beta\}$

Or, $n_1\left\{\frac{AM}{PO} + \frac{AM}{MC}\right\} = n_2\left\{\frac{AM}{MC} - \frac{AM}{MI}\right\}$

Since the aperture is small, ∴ MC = PC, MI = PI

$$\therefore \quad \left\{\frac{n_1}{PO}+\frac{n_1}{PC}\right\} = \left\{\frac{n_2}{PC}-\frac{n_2}{PI}\right\}$$

According to sign convention, PO = $-u$, $PC = R$, PI = v

$$\therefore \quad \left\{\frac{n_1}{-u}+\frac{n_1}{R}\right\} = \left\{\frac{n_2}{R}-\frac{n_2}{v}\right\}$$

Or, $\boldsymbol{\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}}$ This is the required equation.

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14. Derive lens maker's formula.

It is a relation between the focal length of a lens to the refractive index of its material and the radii of curvature of its two surfaces. It is so called because it is used by lens manufacturers to make lenses of particular power from the glass of give refractive index.

Assumptions:

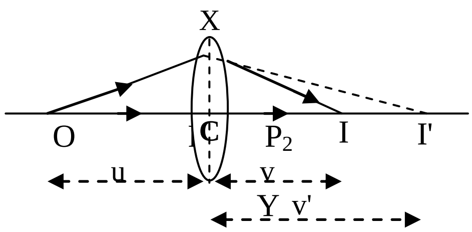
a) The lens is thin so that the distance measured from the poles of the two surfaces of the lens can be taken to be equal to the distances measured from the optical centre.

b) The object is a point object which is situated on the principal axis.

c) The aperture of the lens is small.

d) The incident as well as refracted ray makes small angle with the principal axis.

Consider a thin convex lens made of a material of absolute refractive index n_2 placed in a rarer medium of absolute refractive index n_1. Also, n = $\frac{n_2}{n_1}$ be the refractive index of the material of the lens w.r.t. the medium surrounding it. R_1 and R_2 are the radii of curvature of surfaces XP_1Y and XP_2Y respectively.



For refraction at surface XP_1Y :

'O' is the object and I' is its real image. Using the formula $\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$ we get,

$$\frac{n_2}{v'}-\frac{n_1}{u}=\frac{n_2-n_1}{R_1} \quad \text{........ (i)}$$

For refraction at surface XP_2Y :

I' is the virtual object and I is its real image (final image). Using the formula $\frac{n_1}{v}-\frac{n_2}{u}=\frac{n_1-n_2}{R}$

we get, $$\frac{n_1}{v}-\frac{n_2}{v'}=\frac{n_1-n_2}{R_2} \quad \text{........ (ii)}$$

Adding equations (i) and (ii), we get

$$n_1\left(\frac{1}{v}-\frac{1}{u}\right) = (n_2-n_1)\left\{\frac{1}{R_1}-\frac{1}{R_2}\right\}$$

Or, $$\frac{1}{v}-\frac{1}{u} = \left(\frac{n_2-n_1}{n_1}\right)\left\{\frac{1}{R_1}-\frac{1}{R_2}\right\}$$

Or, $$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\left(\frac{n_2}{n_1}-1\right)\left\{\frac{1}{R_1}-\frac{1}{R_2}\right\}$$

Or, $$\mathbf{P}=\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=(n-1)\left\{\frac{1}{R_1}-\frac{1}{R_2}\right\}$$ This is called lens maker's formula.

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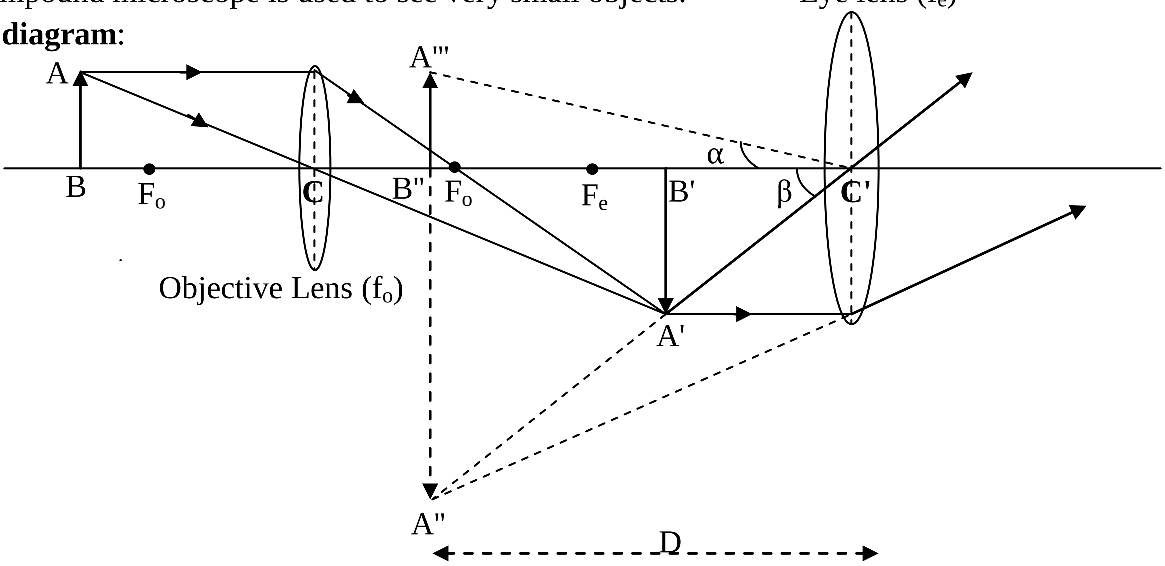
15. With the help of a neat and labeled ray diagram, explain the working of a compound microscope. Also derive an expression for its magnifying power.

A compound microscope is used to see very small objects.

Ray diagram:



Working: - A compound microscope consists of two converging lenses ($f_e > f_o$). The object to be magnified is placed just beyond the focus of the objective lens which forms a real, inverted image. This image is either at the focus or within the focus of the eye lens. The eye lens acts as a simple microscope and forms final image that is virtual, erect and magnified (at D).

Expression for magnifying power (M):

$$M = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{\frac{A''B''}{C'B''}}{\frac{A'''B''}{C'B''}} = \frac{A''B''}{A'''B''} = \left\{\frac{A''B''}{A'B'}\right\}\left\{\frac{A'B'}{A'''B''}\right\} = \left\{\frac{A''B''}{A'B'}\right\}\left\{\frac{A'B'}{AB}\right\} = M_e\,M_o = M_o\,M_e$$

But, $M_o = -\frac{v_o}{u_o}$ and $M_e = 1 + \frac{D}{f_e}$

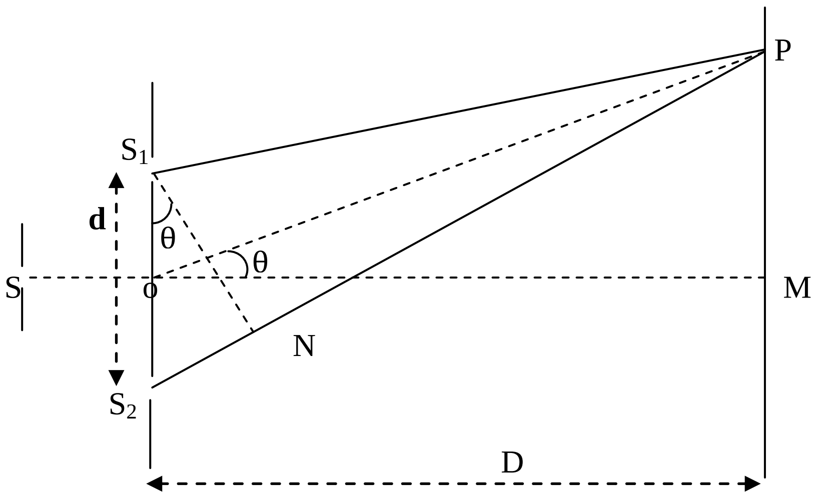
$\therefore$ $\mathbf{M = -\frac{v_o}{u_o}\left\{1 + \frac{D}{f_e}\right\}}$ This is the required relation.

16. Derive a mathematical expression for the width of interference fringes obtained in Young's double slit experiment with the help of a suitable diagram.

Let, 'd' be the distance between two coherent slits S_1 and S_2. The distance between the slits and the screen is 'D'.

The path difference between the two light waves (from two coherent sources S_1 and S_2) arriving at point 'M' on the screen is zero. Thus the point 'M' has maximum intensity.

Consider a point P on the screen at a distance 'y' from 'M'. Draw S_1N perpendicular from S_1 on S_2P.



The path difference between two waves reaching at P from S_1 and S_2 is

$\Delta = S_2P - S_1P \approx S_2N$

As D >> d, therefore, $\angle S_2 S_1N = \theta$ is very small.

$\therefore \angle S_2 S_1N = \angle MOP = \theta$

In $\Delta S_1 S_2N$, $\sin\theta = \frac{S_2N}{S_1S_2}$

In ΔMOP, $\tan\theta = \frac{MP}{OM}$

As 'θ' is very small, $\sin\theta = \tan\theta = \theta$

$\therefore \frac{S_2N}{S_1S_2} = \frac{MP}{OM}$

Or, $S_2N = S_1S_2 \{\frac{MP}{OM}\}$

$\therefore$ path difference, $\Delta = S_2P - S_1P = S_2N = \frac{y\,d}{D}$ ...... (i)

For bright fringe at P, the path difference must be an integral multiple of wavelength of light used.

$\therefore \Delta = n\lambda$ where n = 0,1,2,3,……

Or, $\frac{y\,d}{D} = n\lambda$

$\therefore y = \frac{nD\lambda}{d}$

The above equation gives the position of n^{th} bright fringe from the point M (centre of the screen).

So, $y_n = \frac{nD\lambda}{d}$ …. (ii)

Now, the fringe width (β) is defined as the distance between any two consecutive bright fringes (or dark fringes).

$\therefore \beta = y_{n+1} - y_n = (n + ½)\frac{D\lambda}{d} - (n - ½)\frac{D\lambda}{d} = \frac{D\lambda}{d}(n + ½ - n + ½)$

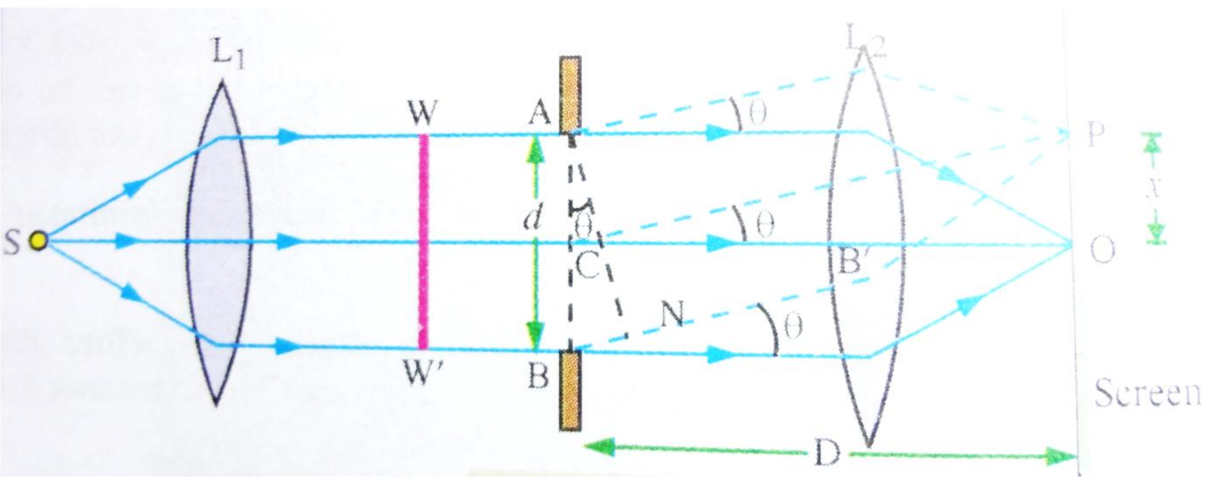
Or, $\boldsymbol{\beta = \frac{D\lambda}{d}}$ which is the required relation.

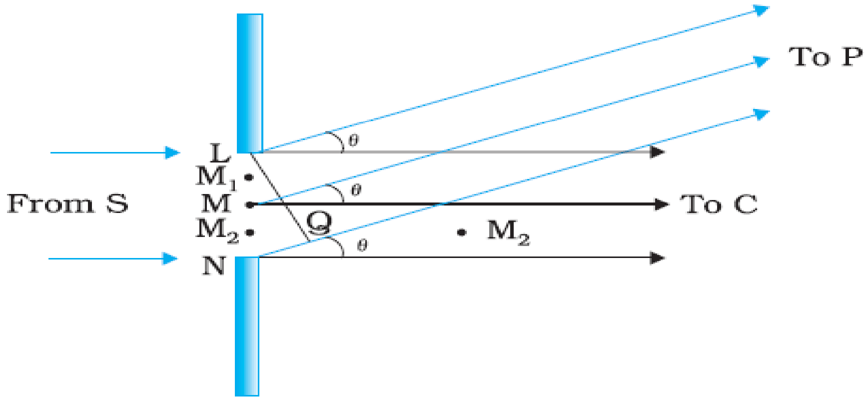
Note: - By using the condition for dark fringes, i.e., Path difference $\Delta = (2n-1)\frac{\lambda}{2}$

where n = 0,1,2,3,….. , we can show that, $\beta = \frac{D\lambda}{d}$. It means fringe width is the same for bright and dark fringes.

17. Describe diffraction of light due to a single slit. Explain the formation of a pattern of fringes on the screen and plot showing variation of intensity with angle θ in single slit diffraction.

Diffraction: - The phenomenon of bending of light around the corners of an obstacles or aperture and entering into the region of geometrical shadow of the obstacle.





Condition for Diffraction – The size of obstacle or aperture should be of the order of the wavelength of the light.

Condition for secondary Minima –

$$a \sin\theta_n = n\lambda$$

a -Size of aperture, n –order of minima, λ –wavelength of light used.

Condition for secondary Maxima –

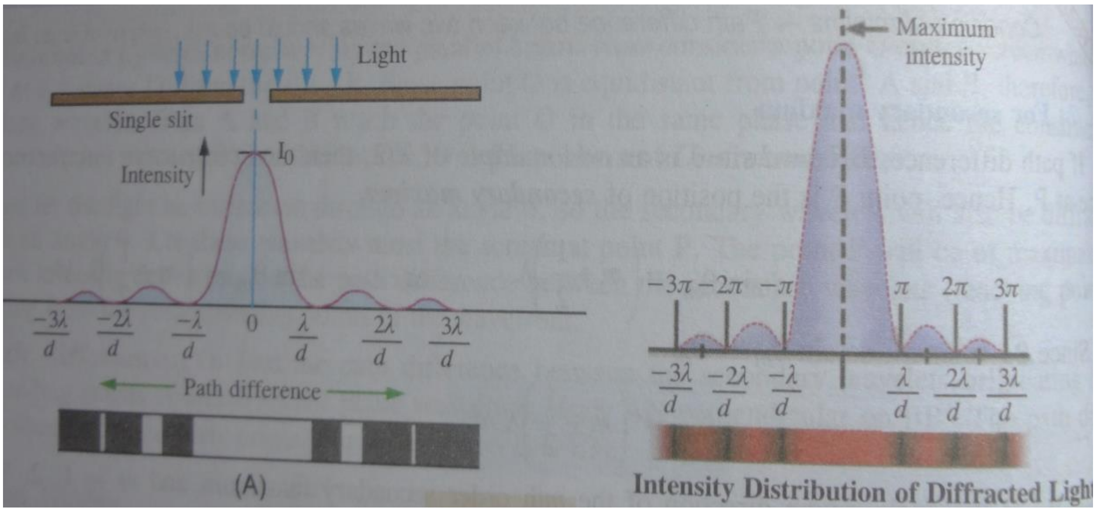
$$a \sin\theta_n = (2n +1)\lambda/2$$

a -Size of aperture, n –order of minima, λ –wavelength of light used.

Width of fringes = $\beta = D\lambda/a$

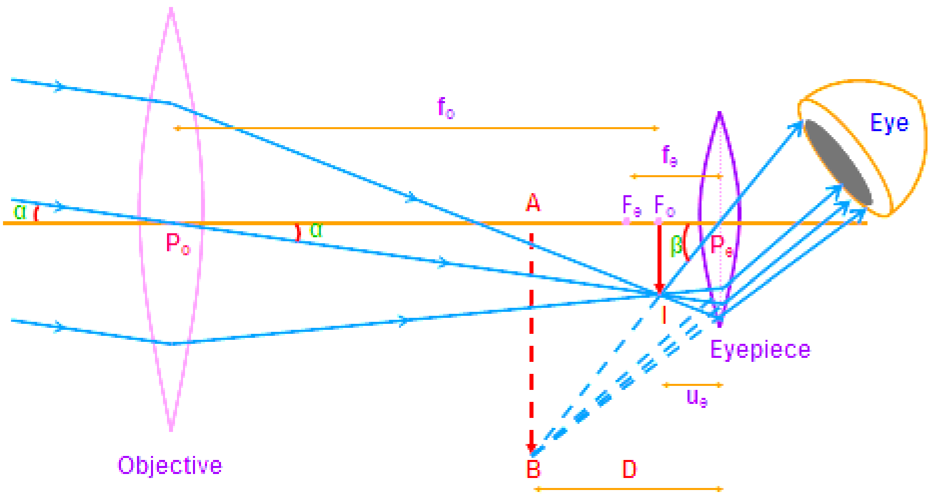
Width of central Bright fringe = $2\beta = 2D\lambda/a$

Graphical Representation of intensity pattern for diffraction –



18. Draw ray diagram for astronomical telescope with image at near point. Derive expression for magnification.

Astronomical Telescope: (Image formed at LDDV)



Angular magnification or magnifying power of a telescope in this case is defined as the ratio of the angle β subtended at the eye by the final image formed at the least distance of distinct vision to the angle α subtended at the eye by the object lying at infinity when seen directly.

$$M = \frac{\beta}{\alpha}$$

Since angles are small,
α = tan α and β = tan β

$$M = \frac{\tan \beta}{\tan \alpha}$$

$$M = \frac{F_o I}{P_e F_o} \Big/ \frac{F_o I}{P_o F_o}$$

$$M = \frac{P_o F_o}{P_e F_o} \quad \text{or} \quad M = \frac{+f_o}{-u_e}$$

Lens Equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$ becomes

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

or $$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

Multiplying by f_o on both sides and rearranging, we get

$$M = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Clearly focal length of objective must be greater than that of the eyepiece for larger magnifying power.

Also, it is to be noted that in this case M is larger than that in normal adjustment position.

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